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Forces on a slender ship advancing near the critical speed in a wide canal

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Employing matched asymptotics, we extend the recent theory of Mei (1986) to study the phenomenon of upstream influence by a slender ship moving near the critical speed. For a special class of channel width and ship slenderness, it is shown that the response on the sea surface is essentially one-dimensional with the wave crests perpendicular to the ship's axis. In particular, solitons are radiated upstream. The hydrodynamic pressure on the ship, as well as the total sinkage force, wave resistance and trimming moment are calculated. These forces are functions of time despite the constant speed of the ship. The sinkage and trim for a ship model fixed on an advancing carriage are computed and show rapid variations across the critical speed as in the reported experiments of Graff, Kracht & Weinblum (1964). Because of the assumed slenderness of the ship, this theory does not predict two-dimensional waves in the wake. Nevertheless, there is crude agreement in the time-averaged hydrodynamic forces between theory and experiment.

1. Introduction

It has been observed in several studies (Thews & Landweber 1935, 1936; Graff, Kracht & Weinblum 1964; Izubuchi & Nagasawa 1937; Kinoshita 1946; Huang, Sibul & Wehausen 1983; Ertekin, Webster & Wehausen 1984) that steady states cannot be attained in a long tank if a ship model is towed at a constant speed U near the natural speed of a long wave, $(gh)^{\frac{1}{2}}$, where h is the still-water depth. Before systematic experiments were available, there had been theories based on the presumption of a steady state. Extending Michell (1898) and Joukowski (1903), Tuck (1966) gave a linearized theory which yields reasonable hydrodynamic forces when the two speeds U and $(gh)^{\frac{1}{2}}$ are substantially different, but unbounded forces when they are close. Nonlinear theories resembling transonic aerodynamics have also been proposed by Maruo (1948), Lea & Feldman (1972) and by Maruo & Tachibana (1981). Their predicted steady responses, although finite, have not been confirmed by experiments. Recent interests in transcritical flows, i.e. $U/(gh)^{\frac{1}{2}} \approx 1$ in a shallow channel have been revived by two independent investigations. For a three-dimensional ship advancing in a channel of finite width, Huang *et al.* (1983) have found experimentally that solitons are periodically emitted upstream. On the other hand, Wu & Wu (1982) have made calculations by using the one-dimensional Boussinesq equations which account for both nonlinearity and dispersion to leading-order, and found upstream solitons for a disturbance spanning uniformly across the channel. Both these studies point to the importance of nonlinearity and dispersion. Akylas (1984) also considered a moving two-dimensional pressure travelling on the free surface but focused attention to the immediate neighbourhood of the critical speed. He showed that the physics can be simply described by an inhomogeneous Korteweg–de Vries (KdV) equation.

One of the interesting aspects observed in the experiments is that three-dimensional disturbance, such as a ship, can generate a two-dimensional flow upstream. The following physical reason for waves to be one-dimensional has been given by Katis & Akylas (1986, private communication). Among all waves radiated from a ship moving at speed U, those which remain stationary with the ship must be in the direction α and have the dimensionless wavenumber k such that $Fkh\cos\alpha = (kh\tanh kh)^{\frac{1}{2}}$. Near the critical speed the waves must be long, i.e. $kh \ll 1$ and $F = 1+O(kh)^2$. It follows that $\cos\alpha = 1+O(kh)^2$ which means that the stationary waves must be in nearly the same direction as the ship speed. It is also intuitively obvious that for these waves to be truly long-crested the channel width must be finite; this was demonstrated experimentally to some extent by Ertekin (1984) and Ertekin *et al.* (1984), who conducted tests in a towing tank whose width was comparable to the ship length. They have shown that the blockage coefficient, which is essentially the area ratio of the ship cross-section to the channel cross-section, is an important parameter. On the theoretical side, Mei (1986) has shown for a vertical strut that the induced transient flow can be two-dimensional everywhere even if the channel is quite wide, as long as the strut is sufficiently slender and its speed is very close to the critical speed $(gh)^{\frac{1}{2}}$. The approximate governing equation is again found to be an inhomogeneous KdV equation. Since the ranges of parameters are quite different, this theory does not predict the three-dimensional wake observed by Ertekin *et al.*, although there is substantial and surprising agreement on the upstream solitons. Recently, Ertekin, Webster & Wehausen (1986) have carried out two-dimensional computations for a rectangular patch of surface pressure whose width is of the same order as the channel width. Their calculations have yielded two-dimensional flow upstream and three-dimensional flow downstream, in qualitative agreement with the measurements. It is as yet unclear how their two-dimensional computation, which is by no means easy, can be modified to account for the three-dimensional neighbourhood of a ship.

In this paper we shall extend the theory of Mei to treat the problem of a ship with a view to calculating the transient forces on the ship. The results are compared with the time-averaged sinkage and trim for a destroyer model measured by Graff *et al.* (1964).

We first summarize the exact basic equations. As is shown in figure 1, the channel width is denoted by $2W$, the ship length by $2L$, the characteristic radius of the ship's cross-section by R_0. We choose a rectangular coordinate system fixed on the waterplane of the ship, so that the x^*-axis coincides with the longitudinal axis of the ship and the centreline of the channel. Symmetry with respect to the (x^*, z^*)-plane is assumed. The potential ϕ^* of the disturbed flow due to the ship must satisfy the Laplace equation in the fluid,

$$\phi^*_{x^*x^*}+\phi^*_{y^*y^*}+\phi^*_{z^*z^*} = 0 \quad (-h < z^* < \zeta^*), \tag{1.1}$$

and the kinematic and dynamic conditions on the free surface at $z^* = \zeta^*$:

$$\phi^*_{z^*} = \zeta^*_{t^*}+(U+\phi^*_{x^*})\,\zeta^*_{x^*}+\phi^*_{y^*}\,\zeta^*_{y^*}, \tag{1.2}$$

$$g\zeta^*+\phi^*_{t^*}+U\phi^*_{x^*}+\tfrac{1}{2}[(\phi^*_{x^*})^2+(\phi^*_{y^*})^2+(\phi^*_{z^*})^2] = 0. \tag{1.3}$$

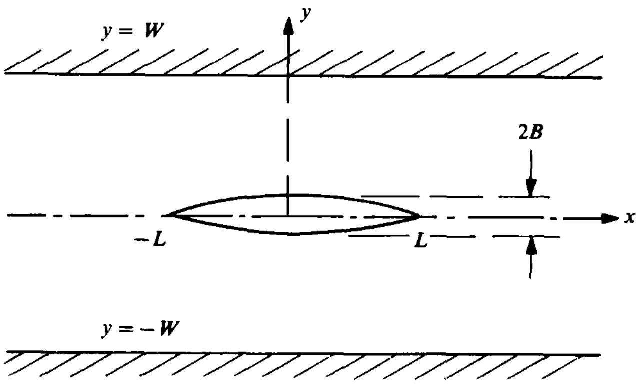

FIGURE 1. Definition sketch of a slender ship moving in a canal.

The normal velocity must vanish on the channel bottom,

$$\phi^*_{z^*} = 0, \quad z^* = -h, \tag{1.4}$$

on the channel bank

$$\phi^*_{y^*} = 0, \quad y^* = W, \tag{1.5}$$

and on the ship hull $r^* = R^*(x^*, \theta)$,

$$\frac{\partial \phi^*}{\partial n^*} = (U + \phi^*_{x^*})\, R^*_{x^*}\left[1 + \left(\frac{R^*_\theta}{R^*}\right)^2\right]^{-\frac{1}{2}}, \tag{1.6}$$

where r^* and θ are polar coordinates in the (y^*, z^*)-plane. Before the initial instant $t^* = 0$, there is no disturbance

$$\xi^* = 0, \quad \phi^* = 0, \quad t^* = 0. \tag{1.7}$$

2. Some results for a vertical strut

To motivate our analysis some key results of Mei (1986) for a strut with vertical walls are needed. Let the hull of the strut be given by $y^* = Y^*(x^*)$ on which the following boundary condition applies,

$$\phi^*_{y^*} = (U + \phi^*_{x^*})\, Y^*_{x^*} \quad (y^* = Y^*(x^*)). \tag{2.1}$$

Two small parameters are introduced which are the measures of nonlinearity and dispersion respectively,

$$\epsilon = \frac{A}{h}, \quad \mu = \frac{h}{L}, \tag{2.2}$$

where A is the typical wave amplitude. Upon introducing the normalization

$$\left.\begin{aligned} \xi^* = A\xi, \quad \phi^* = \frac{gAL}{U}\phi, \quad (x^*, y^*) = L(x, y), \\ z^* = hz, \quad t^* = \frac{L}{(gh)^{\frac{1}{2}}}t, \quad Y^* = BY, \end{aligned}\right\} \tag{2.3}$$

and the following shallow-water expansion,

$$\phi = \phi_0 - \tfrac{1}{2}\mu^2(z+1)^2\,\Delta\phi_0 + \frac{\mu^4}{4!}(z+1)^4\,\Delta\Delta\phi_0 + \ldots, \tag{2.4}$$

we obtain a Boussinesq approximation

$$\Delta\overline{\phi} - \mathrm{D}^2\overline{\phi} - \frac{\epsilon}{2F}\mathrm{D}(\overline{\phi}_x^2 + \overline{\phi}_y^2) - \frac{\epsilon}{F}\nabla\cdot(\mathrm{D}\overline{\phi}\,\nabla\overline{\phi}) + \tfrac{1}{3}\mu^2\mathrm{D}^2\,\Delta\overline{\phi} = O(\epsilon^2, \epsilon\mu^2, \mu^4), \tag{2.5}$$

for the depth-average potential

$$\overline{\phi} = \frac{1}{1+\epsilon\xi}\int_{-1}^{\epsilon\xi}\phi\,\mathrm{d}z. \tag{2.6}$$

The hull condition becomes, to the leading order

$$\overline{\phi}_y \approx \frac{1}{\epsilon}\frac{B}{L}F^2 Y_x \quad \text{on } y = \frac{B}{L}Y, \tag{2.7}$$

where

$$F = \frac{U}{(gh)^{\frac{1}{2}}}, \quad \text{and } \mathrm{D} = \frac{\partial}{\partial t} + F\frac{\partial}{\partial x}. \tag{2.8}$$

Equation (2.5) holds for $\epsilon = O(\mu^2)$ and includes nonlinear and dispersive effects to leading order only. From here on we shall focus our attention on a small neighbourhood of the critical speed,

$$1 - F^2 = 2\alpha\mu^2, \tag{2.9}$$

where $\alpha = O(1)$. We need to rescale time by

$$t = \mu^2\tau, \tag{2.10}$$

in order to account for transient effects. Let the channel width be such that

$$\frac{W}{L} = O(\mu^{-m}), \tag{2.11}$$

and rescale the lateral coordinate by

$$y = \frac{\eta}{\eta_0\mu^m} \quad \text{where } \eta_0 = O(1). \tag{2.12}$$

It then follows from (2.5) and (2.7) that

$$\overline{\phi}_{\eta\eta} = -\left(\frac{\mu^{2-2m}}{\eta_0^2}\right)\left[2\alpha\overline{\phi}_{xx} - 2\overline{\phi}_{\tau x} - 3\frac{\epsilon}{\mu^2}\overline{\phi}_x\,\overline{\phi}_{xx} + \tfrac{1}{3}\overline{\phi}_{xxxx}\right] + O(\mu^{4-2m}), \tag{2.13}$$

and

$$\overline{\phi}_\eta = \frac{BW}{h^2}Y_x = \frac{1}{\epsilon\mu^m\eta_0}\frac{B}{L}Y_x \quad \text{on } y = 0. \tag{2.14}$$

Several situations may arise. If we choose $m = 1$ so that

$$\frac{W}{L} = O(\mu^{-1}), \quad y = \frac{\eta}{\eta_0\mu}, \tag{2.15}$$

then (2.13) is the so called Kadomtsev–Petviashvilli (K–P) equation. A compatible boundary condition on the hull is obtained by letting

$$\frac{BW}{h^2} = O(1), \quad \text{i.e. } \frac{B}{L} = O(\epsilon\mu) = O(\mu^3), \tag{2.16}$$

so that
$$\overline{\phi}_\eta = bY_x, \tag{2.17}$$
where $b = O(1)$. There is no loss of generality by taking $\epsilon = \mu^2$ from here on. This problem governed by (2.13) and (2.17) requires computations involving two space coordinates. On the other hand, Mei (1986) chooses instead
$$m = \tfrac{1}{2} \tag{2.18}$$
so that
$$\frac{W}{L} = O(\mu^{-\frac{1}{2}}), \quad y = \frac{\eta}{\eta_0 \mu^{\frac{1}{2}}}. \tag{2.19a, b}$$
Equations (2.13) and (2.14) can be satisfied only if
$$\beta = \frac{B/W}{\mu^4} = O(1), \quad \text{i.e.}\ \frac{B}{L} = O(\mu^{3.5}). \tag{2.20}$$
Note that B/W is the blockage coefficient. Then to leading order $O(1)$, the free-surface displacement
$$\xi = -\overline{\phi}_x + O(\mu), \tag{2.21}$$
can be shown to satisfy the inhomogeneous KdV equation
$$\xi_\tau - \alpha\xi_x - \tfrac{3}{2}\xi\xi_x - \tfrac{1}{6}\xi_{xxx} = \tfrac{1}{2}\beta Y_x. \tag{2.22}$$
The mathematical problem depends only on one space coordinate. Numerical results of Mei show that the upstream solitons are in surprising agreement with the experiments of Ertekin *et al.* (1984) which fall in the range $W/L = O(1)$ and $B/L = O(\mu^2)$, although the observed free surface is highly two-dimensional in the wake.

It may be readily shown that the same result (2.22) holds for $0 \leqslant m \leqslant \frac{1}{2}$. In particular for $m = 0$ we must choose
$$\frac{BW}{h^2} = O(\mu^2), \tag{2.23}$$
in order that (2.13) and (2.14) are consistent at the first two orders. This means that
$$\frac{W}{L} = O(1), \quad \frac{B}{L} = \frac{B}{W} = O(\mu^4), \tag{2.24a, b}$$
i.e. the strut is very slender.

To treat the case studied experimentally by Ertekin *et al.* (1984) it is necessary to abolish the Boussinesq approximation (2.5) and allow nonlinearity to be much larger than dispersion (e.g. $\epsilon = O(\mu)$). This would require a greater computational effort involving two-space dimensions as in Ertekin *et al.*, but is not pursued here.

3. Plan for asymptotic analysis for a slender ship

We shall only treat cases where the far-field of the ship is one-dimensional to leading order. For this, the ship must have a blockage coefficient of the magnitude of $O(\mu^4)$ (cf. (2.24*b*)). The channel width can be within the range $O(1) \leqslant W/L \leqslant O(\mu^{-\frac{1}{2}})$. For the sake of simplicity only the case $W/L = O(1)$ is explicitly treated in this paper. Thus we assume the characteristic cross-sectional area R_0^2 to be of the order of
$$R_0^2 = O(Bh) = O(\mu^4 Wh) = O(\mu^5 L^2). \tag{3.1}$$

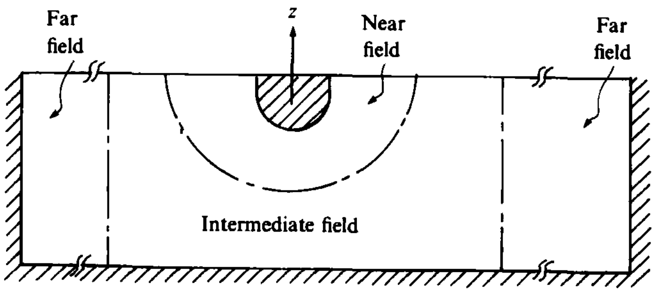

FIGURE 2. Definition sketch of different flow regions.

R_0 will be referred to as the effective half-beam of the ship. The slenderness ratio must then be

$$\delta = \frac{R_0}{L} = O(\mu^{2.5}), \tag{3.2}$$

which implies that the beam-to-depth ratio

$$\frac{R_0}{h} = O\left(\frac{\delta}{\mu}\right) = O(\mu^{1.5}), \tag{3.3}$$

is also small. Because of these different length ratios, it is advantageous to separate the channel cross-section into three regions:

(i) the far field: $|x| < \infty, y = O(W) = O(L), z = O(h) = O(\mu L)$,

(ii) the intermediate field: $|x| = O(L), (|y|, |z|) = O(h) = O(\mu L)$,

(iii) the near field: $|x| = O(L), (|y|, |z|) = O(R_0) = O(\delta L) = O(\mu^{2.5} L)$,

as sketched in figure 2. Heuristically the near field is not directly affected by either the bottom or the channel banks. In the intermediate field, the channel bottom is directly felt but the detailed geometry of the ship or the banks is unimportant. The ship must appear merely as a line distribution of two-dimensional sources emitting fluid laterally. In the far field, the banks and the bottom directly affect the formation of waves which are forced by the fluid flux passing through the intermediate region from the ship axis. It may be noted that in the asymptotic theory of Tuck (1966), which the present work partially resembles, there is no intermediate region; the near and far fields can be matched directly.

We now carry out the details of these fields which will be matched in order to obtain the final governing equations.

4. The far field

Instead of applying the analysis of Mei (1986), it is more convenient for matching purposes to carry out the formalism of the far-field analysis as in Tuck (1966). First we adopt the normalization as in Mei.

$$\left.\begin{aligned} x^* &= Lx, \quad y^* = Wy, \\ z^* &= hz, \quad t^* = \frac{Lt}{\mu^2 (gh)^{\frac{1}{2}}}, \end{aligned}\right\} \tag{4.1}$$

for the independent variables, and

$$\xi^* = L\mu^3\xi, \quad \phi^* = \frac{gL^2}{U}\mu^3\phi, \tag{4.2}$$

for the dependent variables. Consistent with (2.24) we let

$$\eta_0^{-1} = \frac{W}{L} = O(1). \tag{4.3}$$

In terms of these the Laplace equation reads:

$$\mu^2(\phi_{xx}+\eta_0^2\phi_{yy})+\phi_{zz} = 0 \quad (-1 < z < \mu^2\xi). \tag{4.4}$$

The boundary conditions on the free surface become

$$\begin{gathered}\phi_z = \mu^2[(1-2\alpha\mu^2)\,\xi_x+\mu^2(1-2\alpha\mu^2)^{\frac{1}{2}}\,\xi_t+\mu^2(\phi_x\,\xi_x+\eta_0^2\,\phi_y\,\xi_y)]\\ (1-2\alpha\mu^2)\,(\xi+\phi_x)+(1-2\alpha\mu^2)^{\frac{1}{2}}\mu^2\phi_t+\tfrac{1}{2}[\mu^2(\phi_x^2+\eta_0^2\,\phi_y^2)+\phi_z^2] = 0 \quad (z = \mu^2\xi).\end{gathered} \tag{4.5}$$

On the channel bottom and banks we have

$$\phi_z = 0 \quad (z = -1), \tag{4.6}$$

and

$$\phi_y = 0 \quad (y = 1). \tag{4.7}$$

Substituting the expansions

$$\phi = \phi^{(0)}+\mu^2\phi^{(2)}+\ldots, \tag{4.8}$$

$$\xi = \xi^{(0)}+\mu^2\xi^{(2)}+\ldots, \tag{4.9}$$

we obtain from (4.4)

$$\left.\begin{aligned}\phi_{zz}^{(0)} &= 0,\\ \phi_{zz}^{(2)} &= -(\phi_{xx}^{(0)}+\eta_0^2\,\phi_{yy}^{(0)}).\end{aligned}\right\} \tag{4.10}$$

Terms of order $\mu = \mu\phi^{(1)}$ and $\mu\xi^{(1)}$ satisfy the same conditions as $\phi^{(0)}$ and $\xi^{(0)}$ and are therefore omitted. From (4.6) and (4.7) we have

$$\phi_z^{(0)} = \phi_z^{(2)} = 0 \quad (z = -1), \tag{4.11}$$

$$\phi_y^{(0)} = \phi_y^{(2)} = 0 \quad (y = 1). \tag{4.12}$$

After Taylor expansion, the kinematic free-surface condition gives

$$\left.\begin{aligned}\phi_z^{(0)} &= 0\\ \phi_z^{(2)} &= \xi_x^{(0)}\end{aligned}\right\} \quad (z = 0). \tag{4.13}$$

Similarly, the dynamic free-surface condition gives

$$\left.\begin{aligned}\xi^{(0)} &= -\phi_x^{(0)}\\ \xi^{(2)} &= -\phi_x^{(2)}-\tfrac{1}{2}(\phi_x^{(0)^2}+\eta_0^2\,\phi_y^{(0)^2})-\phi_t^{(0)}\end{aligned}\right\} \quad (z = 0). \tag{4.14}$$

Making use of (4.11) and (4.13), (4.10) can be integrated vertically to give

$$\left.\begin{aligned}\phi^{(0)} &= \psi^{(0)}(x,y,t), \quad \phi^{(1)} = \psi^{(1)}(x,y,t),\\ \phi^{(2)} &= \psi^{(2)}(x,y,t)-\tfrac{1}{2}(z+1)^2\,(\psi_{xx}^{(0)}+\eta_0^2\,\psi_{yy}^{(0)}).\end{aligned}\right\} \tag{4.15}$$

Substituting (4.15) into (4.13) order by order, we obtain

$$\left.\begin{aligned} \xi_x^{(0)} &= -(\psi_{xx}^{(0)}+\eta_0^2\psi_{yy}^{(0)}), \\ \xi_x^{(2)} &= -(\psi_{xx}^{(2)}+\eta_0^2\psi_{yy}^{(2)})+\tfrac{1}{6}\psi_{xxxx}^{(0)}-\xi^{(0)}(\psi_{xx}^{(0)}+\eta_0^2\psi_{yy}^{(0)})+2\alpha\xi_x^{(0)}-\xi_t^{(0)}-\psi_x^{(0)}\xi_x^{(0)}. \end{aligned}\right\} \tag{4.16}$$

Similar manipulations of the dynamic condition yields

$$\left.\begin{aligned} \xi^{(0)} &= -\psi_x^{(0)}, \\ \xi^{(2)} &= -\psi_x^{(2)}+\tfrac{1}{2}(\psi_{xxx}^{(0)}+\eta_0^2\psi_{xyy}^{(0)})-\tfrac{1}{2}(\psi_x^{(0)^2}+\eta_0^2\psi_y^{(0)^2})-\psi_t^{(0)}. \end{aligned}\right\} \tag{4.17}$$

We now substitute (4.17) into (4.16) order by order. Non-trivial information begins at $O(\mu^2)$;

$$\left.\begin{aligned} \psi_{yy}^{(0)} &= 0, \\ \tfrac{1}{2}\psi_{xxxx}^{(0)}-\psi_x^{(0)}\psi_{xx}^{(0)}-\psi_{xt}^{(0)} &= -\eta_0^2\psi_{yy}^{(2)}+\tfrac{1}{6}\psi_{xxxx}^{(0)}-\xi^{(0)}\psi_{xx}^{(0)}+2\alpha\xi_x^{(0)}-\xi_t^{(0)}-\psi_x^{(0)}\xi_x^{(0)}. \end{aligned}\right\} \tag{4.18}$$

With the help of (4.14), the last equation above can be written as:

$$\xi_t^{(0)}-\alpha\xi_x^{(0)}-\tfrac{3}{2}\xi^{(0)}\xi_x^{(0)}-\tfrac{1}{6}\xi_{xxx}^{(0)} = -\tfrac{1}{2}\eta_0^2\psi_{yy}^{(2)}(x,y,t). \tag{4.19}$$

Since the boundary condition on the channel bank implies

$$\psi_y^{(0)} = \psi_y^{(1)} = \psi_y^{(2)} = 0 \quad (y=1), \tag{4.20}$$

$\psi^{(0)}$ and $\psi^{(1)}$ must be independent of y. Integrating (4.19) from $y=0$ to 1 then gives

$$\xi_t^{(0)}-\alpha\xi_x^{(0)}-\tfrac{3}{2}\xi^{(0)}\xi_x^{(0)}-\tfrac{1}{6}\xi_{xxx}^{(0)} = \tfrac{1}{2}\eta_0^2\psi_y^{(2)}(x,0,t). \tag{4.21}$$

Equation (4.21) is an inhomogeneous KdV equation similar to that of Mei for a strut. The right-hand side clearly represents a transverse flux and will be determined by matching with the intermediate field. For this purpose we need the following inner expansion of ϕ for small y:

$$\phi = \psi^{(0)}(x,t)+\mu^2[\psi^{(2)}(x,0,t)-\tfrac{1}{2}(z+1)^2\psi_{xx}^{(0)}(x,t)+y\psi_y^{(2)}(x,0,t)]+O(\mu^3). \tag{4.22}$$

5. The near field

Let us introduce the normalized near-field variables

$$y^* = R_0\bar{y}, \quad z^* = R_0\bar{z}, \quad r^* = R_0\bar{r}, \quad R^* = R_0\bar{R}, \tag{5.1}$$

but retain the rest of the normalization defined in (4.1) and (4.2). The Laplace equation now gives

$$\delta^2\phi_{xx}+\phi_{\bar{y}\bar{y}}+\phi_{\bar{z}\bar{z}} = 0 \quad \left(-\infty < \bar{z} < \frac{\mu^3}{\delta}\xi\right). \tag{5.2}$$

On the free surface at $\bar{z} = (\mu^3/\delta)\,\xi$, we have

$$\phi_{\bar{z}} = \mu^3\delta(1-2\alpha\mu^2)^{\frac{1}{2}}\xi_t+\mu\delta(1-2\alpha\mu^2)\,\xi_x+\mu^3\delta\phi_x\,\xi_x+\left(\frac{\mu^3}{\delta}\right)\phi_{\bar{y}}\,\xi_{\bar{y}}. \tag{5.3}$$

On the ship hull $\bar{r} = \bar{R}(x,\theta)$, condition (1.6) may be written

$$\frac{\partial\phi}{\partial\bar{n}} = \left(\frac{\delta}{\mu}\right)^2(1-2\alpha\mu^2+\mu^2\phi_x)\,\bar{R}_x\left[1+\left(\frac{\bar{R}_\theta}{\bar{R}}\right)^2\right]^{-\frac{1}{2}}. \tag{5.4}$$

In addition we impose $$\phi_{\bar{z}} \to 0 \quad \text{as } \bar{z} \to -\infty. \tag{5.5}$$

Note from (3.2) that $$\frac{\mu^3}{\delta} = O(\mu^{0.5}), \quad \left(\frac{\delta}{\mu}\right)^2 = O(\mu^3). \tag{5.6}$$

We therefore try the following expansions:

$$\left.\begin{aligned} \phi &= \bar{\phi}^{(0)} + \mu^2 \bar{\phi}^{(2)} + \left(\frac{\delta}{\mu}\right)^2 \bar{\phi}^{(3)} + \ldots, \\ \xi &= \bar{\xi}^{(0)} + \mu^2 \bar{\xi}^{(2)} + \left(\frac{\delta}{\mu}\right)^2 \bar{\xi}^{(3)} + \ldots. \end{aligned}\right\} \tag{5.7}$$

From (5.2), (5.4) and (5.5) the following perturbation equations are easily obtained:

$$\phi^{(n)}_{\bar{y}\bar{y}} + \phi^{(n)}_{\bar{z}\bar{z}} = 0 \quad (n = 0, 2, 3), \tag{5.8}$$

$$\left.\begin{aligned} \frac{\partial \bar{\phi}^{(n)}}{\partial \bar{n}} &= 0 \quad (n = 0, 2) \\ \frac{\partial \bar{\phi}^{(3)}}{\partial \bar{n}} &= \bar{R}_x \left[1 + \left(\frac{\bar{R}_\theta}{\bar{R}}\right)^2\right]^{-\frac{1}{2}} \end{aligned}\right\} \quad \bar{r} = \bar{R}(x, \theta), \tag{5.9}$$

$$\bar{\phi}^{(n)}_{\bar{z}} \downarrow 0 \quad \bar{z} \downarrow -\infty \quad (n = 0, 2, 3). \tag{5.10}$$

On the free surface the kinematic condition (5.3) can be approximated to leading order by:

$$\bar{\phi}^{(0)}_{\bar{z}} = 0 \quad (\bar{z} = 0). \tag{5.11}$$

Clearly $\bar{\phi}^{(0)}$ can only depend on x and t. This fact can be utilized to simplify the Taylor expansion of (5.3) to higher orders, with the following results valid on the free surface

$$\bar{\phi}^{(n)}_{\bar{z}} = 0 \quad (n = 2, 3). \tag{5.12}$$

The homogeneous Neuman problems imply that

$$\bar{\phi}^{(n)} = f^{(n)}(x, t) \quad (n = 0, 2). \tag{5.13}$$

The inhomogeneous Neuman problem for $\bar{\phi}^{(3)}$ has the following general solution

$$\bar{\phi}^{(3)} = f^{(3)}(x, t) + \bar{\phi}_{\mathrm{p}}(x, \bar{y}, \bar{z}, t), \tag{5.14}$$

where $\bar{\phi}_{\mathrm{p}}$ is the particular solution the details of which depend on the ship geometry, and will contribute to the pressure distribution at $O(\mu^2)$. Its outer approximation for large r will prove to be more important and may be written:

$$\bar{\phi}_{\mathrm{p}} = \frac{1}{\pi} q(x, t) \ln \bar{r} + c(x, t), \tag{5.15}$$

where c may be regarded as a part of $f^{(3)}$. In physical variables we must have:

$$\phi^*_{\mathrm{p}} = \frac{1}{\pi} U \frac{\partial S^*}{\partial x^*} \ln \frac{r^*}{h} = \frac{U}{\pi} \frac{S^*_0}{L} S_x \ln \frac{R_0}{h} \bar{r} = \frac{gLh\delta^2}{U} \frac{q}{\pi} \ln \bar{r} + c, \tag{5.16}$$

where S^* is the cross-sectional area, and S^*_0 its maximum, of the ship below the water plane. Defining the blockage coefficient by

$$S_{\mathrm{B}} = \frac{S^*_0}{2Wh}, \tag{5.17}$$

and using (4.3), we get

$$q = \frac{2}{\eta_0}\frac{\mu}{\delta^2}S_B S_x, \tag{5.18}$$

and

$$c = \frac{q}{\pi}\ln\frac{R_0}{h} = \frac{q}{\pi}\ln\frac{\delta}{\mu}. \tag{5.19}$$

In order to match with the intermediate field, we need the outer expansion of the near field for large $\bar{r}$:

$$\phi = f^{(0)} + \mu^2 f^{(2)} + \left(\frac{\delta}{\mu}\right)^2\left[f^{(3)} + \frac{q}{\pi}\ln\left(\frac{\delta}{\mu}\bar{r}\right)\right] + \dots. \tag{5.20}$$

6. The intermediate field

Here the proper scale for y and r is h, hence we introduce the intermediate coordinates $\tilde{y}$ and $\tilde{r}$:

$$y^* = h\tilde{y}, \quad r^* = h\tilde{r}, \tag{6.1}$$

and keep all other normalized variables defined in (4.1) and (4.2). The dimensionless equations are

$$\mu^2\phi_{xx} + \phi_{\tilde{y}\tilde{y}} + \phi_{zz} = 0 \quad (-1 < z < \mu^2\xi), \tag{6.2}$$

$$\phi_z = \mu^4(1-2\alpha\mu^2)^{\frac{1}{2}}\xi_t + \mu^2(1-2\alpha\mu^2)\,\xi_x + \mu^4\phi_x\xi_x + \mu^2\phi_{\tilde{y}}\,\xi_{\tilde{y}} \quad (z = \mu^2\xi), \tag{6.3}$$

$$(1-2\alpha\mu^2)(\xi+\phi_x) + \mu^2(1-2\alpha\mu^2)^{\frac{1}{2}}\phi_t + \tfrac{1}{2}(\mu^2\phi_x^2 + \phi_{\tilde{y}}^2 + \phi_z^2) = 0 \quad (z = \mu^2\xi), \tag{6.4}$$

$$\phi_z = 0 \quad (z = -1). \tag{6.5}$$

In anticipation of matching with the near field, we introduce expansions in the form of (5.7), with $\bar{\phi}^{(n)}$ and $\bar{\xi}^{(n)}$ replaced by $\tilde{\phi}^{(n)}$ and $\tilde{\xi}^{(n)}$, respectively, it then follows from (6.2) and (6.5) that

$$\left.\begin{aligned} \tilde{\phi}^{(0)}_{\tilde{y}\tilde{y}} + \tilde{\phi}^{(0)}_{zz} &= 0, \\ \tilde{\phi}^{(2)}_{\tilde{y}\tilde{y}} + \tilde{\phi}^{(2)}_{zz} &= -\tilde{\phi}^{(0)}_{xx}, \end{aligned}\right\} \tag{6.6}$$

and

$$\tilde{\phi}^{(n)}_z = 0 \quad (n = 0, 2, 3; \quad z = -1). \tag{6.7}$$

From the Taylor expansions of the two conditions on the free surface, we get

$$\left.\begin{aligned} \tilde{\phi}^{(0)}_z &= 0 \\ \tilde{\phi}^{(2)}_z &= \tilde{\xi}^{(0)}_x \end{aligned}\right\} \quad (z = 0), \tag{6.8}$$

and

$$\tilde{\xi}^{(0)} = -\tilde{\phi}^{(0)}_x. \tag{6.9}$$

It is easy to see that

$$\left.\begin{aligned} \tilde{\phi}^{(0)} &= \tilde{f}^{(0)}(x,t), \\ \tilde{\phi}^{(2)} &= \tilde{f}^{(2)}(x,t) - \tfrac{1}{2}(z+1)^2\tilde{f}^{(0)}_{xx}, \\ \tilde{\phi}^{(3)} &= \tilde{f}^{(3)}(x,t) + \tilde{\phi}_p. \end{aligned}\right\} \tag{6.10}$$

The particular solution $\tilde{\phi}_p$ corresponds to the response to a line source at the origin, hence has the following behaviour

$$\tilde{\phi}_p = \frac{1}{\pi}\tilde{q}(x,t)\ln\tilde{r} = \frac{1}{\pi}\tilde{q}\ln\left(\frac{\delta}{\mu}\tilde{r}\right) \quad (\tilde{r} \ll 1), \tag{6.11}$$

near the origin. The inner expansion of the intermediate field is

$$\phi = \tilde{f}^{(0)} + \mu^2(\tilde{f}^{(2)} - \tfrac{1}{2}\tilde{f}^{(0)}_{xx}) + \left(\frac{\delta}{\mu}\right)^2 \left(\tilde{f}^{(3)} + \frac{1}{\pi}\tilde{q}\ln\tilde{r}\right) + \dots. \tag{6.12}$$

In order to match this with the outer expansion of the near field given by (5.20), we must have

$$\left.\begin{aligned} q = \tilde{q} \quad f^{(n)} &= \tilde{f}^{(n)} \quad (n = 0, 1), \\ f^{(2)} = \tilde{f}^{(2)} - \tfrac{1}{2}\tilde{f}^{(0)}_{xx} \quad f^{(3)} &= \tilde{f}^{(3)}. \end{aligned}\right\} \tag{6.13}$$

To seek the outer expansion of the intermediate field, we invoke mass conservation. Then the approximation of $\tilde{\phi}_{\mathrm{p}}$ for large $\tilde{y}$ must be

$$\tilde{\phi}_{\mathrm{p}} \to \tfrac{1}{2}q\tilde{y} \quad (\tilde{y} \gg 1), \tag{6.14}$$

so that the outer expansion of the intermediate field is, in terms of the far-field variables,

$$\phi = \tilde{f}^{(0)} + \mu\tilde{f}^{(1)} + \mu^2[\tilde{f}^{(2)} - \tfrac{1}{2}(z+1)^2\tilde{f}^{(0)}_{xx}] + \left(\frac{\delta}{\mu}\right)^2 \left[(\tilde{f}^{(3)} + \frac{1}{2}\frac{qy}{\mu\eta_0}\right], \tag{6.15}$$

where use has been made of (4.3). Now we match (6.15) with (4.22), taking (5.6) into account, then

$$\left.\begin{aligned} \psi^{(0)} &= \tilde{f}^{(0)}, \\ \psi^{(2)}_y(x, 0, t) &= \frac{1}{2\eta_0}\frac{\delta^2}{\mu^5}q(x,t) = \frac{1}{\eta_0^2}\frac{S_{\mathrm{B}}}{\mu^4}S_x(x,t). \end{aligned}\right\} \tag{6.16}$$

Combining (6.16) with (4.21) we finally obtain the governing equation for $\xi^{(0)}$

$$\xi^{(0)}_t - \alpha\xi^{(0)}_x - \tfrac{3}{2}\xi^{(0)}\xi^{(0)}_x - \tfrac{1}{6}\xi^{(0)}_{xxx} = \frac{1}{2}\frac{S_{\mathrm{B}}}{\mu^4}S_x. \tag{6.17}$$

It is an inhomogeneous KdV equation similar to that of Mei for a strut, in which case the right-hand side must be replaced by

$$\frac{B/W}{2\mu^4}Y_x, \tag{6.18}$$

where $2BY$ is the local beam of the strut.

Once $\xi^{(0)}$ is calculated, $\psi^{(0)} = \tilde{f}^{(0)} = f^{(0)}$ is known. Thus, the potential and the dynamic pressure near the ship are found to the leading order.

7. Forces and the implied ship displacements

The Bernoulli equation,

$$-\frac{p^*}{\rho} = \phi^*_{t^*} + U\phi^*_{x^*} + \tfrac{1}{2}|\nabla^*\phi^*|^2, \tag{7.1}$$

can be expressed in the normalized near-field variables as

$$p = \frac{p^*}{\rho g h} = -\left[\frac{\mu^4}{F}\phi_t + \mu^2\phi_x + \frac{\mu^4}{F^2}\phi_x^2 + \frac{1}{\mu F^2}(\phi_{\bar{y}}^2 + \phi_{\bar{z}}^2)\right]. \tag{7.2}$$

Substituting the expansion (5.7), we find

$$p = -\mu^2\overline{\phi}^{(0)}_x + O(\mu^3), \tag{7.3}$$

which is uniform in the cross-sectional plane at any x, to the order of accuracy indicated. The total force on the ship is then

$$-\iint p^{*}\boldsymbol{n}\,\mathrm{d}S^{*} = \boldsymbol{e}_x \int_{-L}^{L} p^{*}\frac{\partial S^{*}}{\partial x^{*}}\,\mathrm{d}x^{*} + 2\boldsymbol{e}_z \int_{-L}^{L} p^{*}\,Y^{*}(x^{*},t^{*})\,\mathrm{d}x^{*}, \tag{7.4}$$

where Y^* is the half beam of the ship at the waterplane. Normalized by the displacement $\rho g V$ the dimensionless wave resistance is given by

$$R_{\mathrm{w}} = \frac{R_{\mathrm{w}}^{*}}{\rho g V} = \frac{1}{\rho g V}\int_{-L}^{L} p^{*}\frac{\partial S^{*}}{\partial x^{*}}\,\mathrm{d}x^{*} = \frac{\mu^{3}}{2C_{\mathrm{p}}}\int_{-1}^{1} \xi^{(0)} S_x\,\mathrm{d}x, \tag{7.5}$$

where

$$C_{\mathrm{p}} = \frac{V}{2LS_0^{*}}. \tag{7.6}$$

is called the prismatic coefficient. The dimensionless vertical lift force is

$$F_z = \frac{F_z^{*}}{\rho g L A_{\mathrm{w}}} = \frac{2}{\rho g L A_{\mathrm{w}}}\int_{-L}^{L} p^{*}\,Y^{*}\,\mathrm{d}x^{*} = \frac{\mu^{3}}{2C_{\mathrm{w}}}\int_{-1}^{1} \xi^{(0)}\,Y\,\mathrm{d}z, \tag{7.7}$$

where A_{w} is the waterplane area and

$$C_{\mathrm{w}} = \frac{A_{\mathrm{w}}}{4BL} \tag{7.8}$$

is called the waterplane area coefficient. The dimensionless trimming moment M, defined to be positive if it tends to lift the bow, is

$$M = \frac{M^{*}}{\rho g L^{2} A_{\mathrm{w}}} = -\frac{\mu^{3}}{2C_{\mathrm{w}}}\int_{-1}^{1} x\,Y\xi^{(0)}\,\mathrm{d}x. \tag{7.9}$$

The sinkage s^* and trim θ_{T} are defined by the hydrostatic relations (Tuck 1966)

$$-F_z^{*} = 2\rho g\int_{-L}^{L} (s^{*}+x^{*}\theta_{\mathrm{T}})\,Y^{*}\,\mathrm{d}x^{*}, \tag{7.10}$$

and

$$M^{*} = 2\rho g\int_{-L}^{L} x^{*}(s^{*}+x^{*}\theta_{\mathrm{T}})\,Y^{*}\,\mathrm{d}x^{*}. \tag{7.11}$$

Let $s = s^*/L$ be the dimensionless sinkage. Equations (7.10) and (7.11) may then be written in normalized forms:

$$\left.\begin{aligned} -F_z &= s + l\theta_{\mathrm{T}}, \\ M &= ls + r_{\mathrm{g}}\,\theta_{\mathrm{T}}, \end{aligned}\right\} \tag{7.12}$$

where l and r_{g} are respectively the longitudinal centre of flotation and radius of gyration:

$$l = \frac{2}{A_{\mathrm{w}}L}\int_{-L}^{L} x^{*}\,Y^{*}\,\mathrm{d}x^{*} \quad r_{\mathrm{g}} = \frac{2}{A_{\mathrm{w}}L^{2}}\int_{-L}^{L} x^{*2}\,Y^{*}\,\mathrm{d}x^{*}. \tag{7.13}$$

Finally the dimensionless sinkage and trim are

$$\left.\begin{aligned} s &= -\frac{r_{\mathrm{g}}F_z + lM}{r_{\mathrm{g}} - l^{2}}, \\ \theta_{\mathrm{T}} &= \frac{lF_z + M}{r_{\mathrm{g}} - l^{2}}. \end{aligned}\right\} \tag{7.14}$$

These can be calculated from (7.7) and (7.9). Note that they do not represent the actual sinkage and trim of an unconstrained ship.

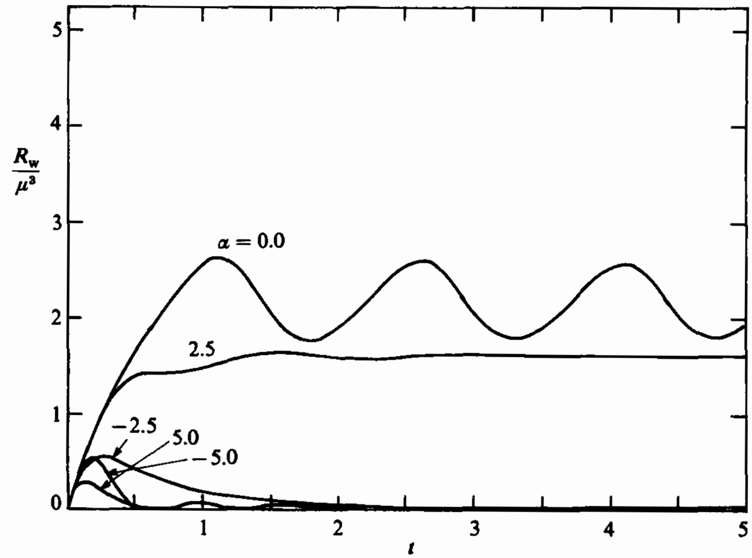

FIGURE 3. Evolution of wave resistance on a slender ship, $\beta = 5$.

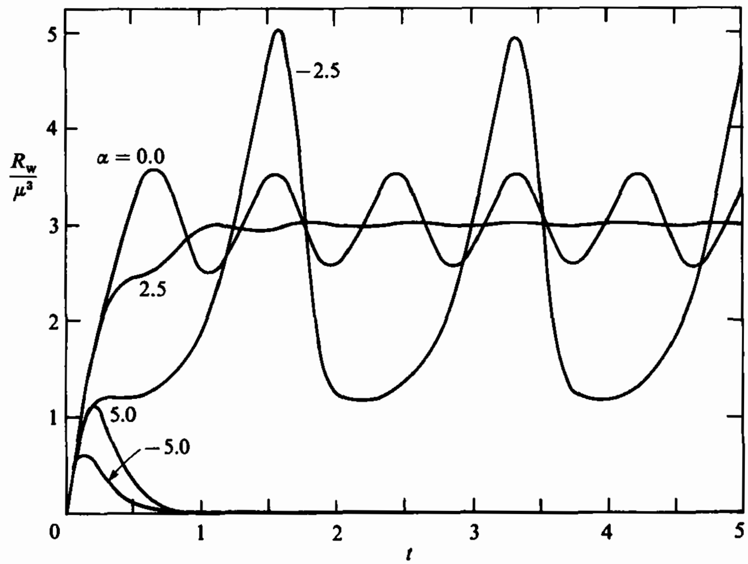

FIGURE 4. Evolution of wave resistance on a slender ship, $\beta = 10$.

8. Numerical results for a parabolic ship

To see the dependence of the forces on the parameters α and β defined respectively in (2.9) and (2.20), we shall consider a simple ship whose cross-sectional area varies parabolically along the axis

$$S(x) = 1 - x^2 \quad (|x| \leqslant 1).$$

As in Mei (1986), (6.17) is solved by the explicit finite difference scheme of Johnson (1972). To satisfy his criteria for stability the steps in x and t are chosen to be 0.1 and 0.0005, respectively. The wave resistance, vertical lift and trimming moments

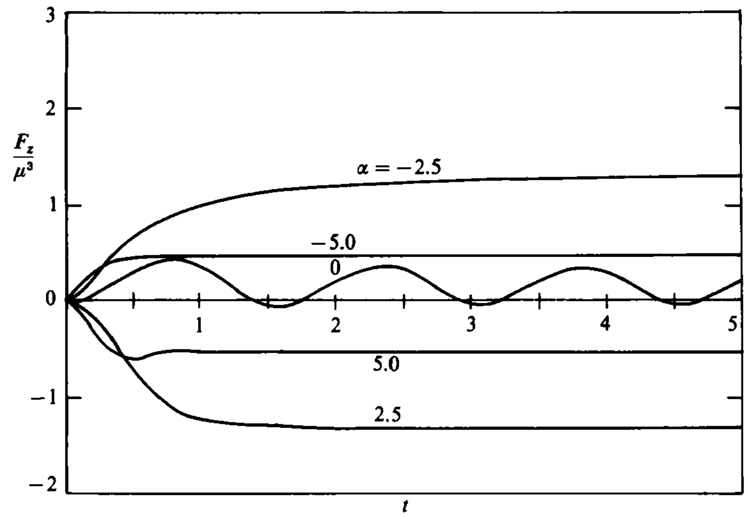

FIGURE 5. Evolution of vertical lift on a slender ship, $\beta = 5$.

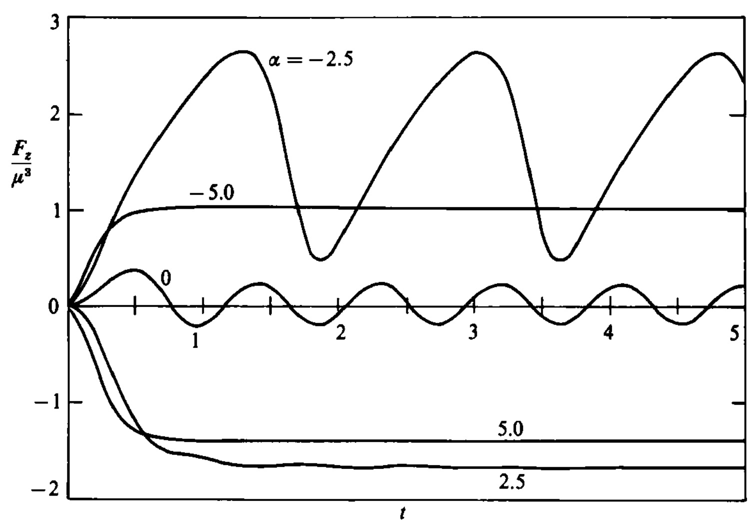

FIGURE 6. Evolution of vertical lift on a slender ship, $\beta = 10$.

are then computed to leading order from (7.5) to (7.9) and are plotted as functions of time for two ships ($\beta = 5$ and 10) and five speeds: $\alpha = 5.0, 2.5$ (subcritical), $\alpha = 0$ (critical) and $\alpha = -2.5, -5.0$ (supercritical). In Mei (1986) the wave profiles have been reported for the same set of speeds for $\beta = 10.4$ and the soliton amplitudes have been given for $\beta = 5$ and 10.4. It was shown there that for a fixed β there was only one small soliton for the low subcritical speed $\alpha = 5.0$. The number and size of upstream solitons increase with the ship speed (hence with $-\alpha$), up to a supercritical threshold beyond which there are neither solitons upstream nor shipbound waves downstream.

As shown in figures 3 and 4, the wave resistance for either ship rises initially to a maximum, corresponding to the emission of only one soliton, and diminishes in time to a small steady value due to the running away of the soliton and the rather small

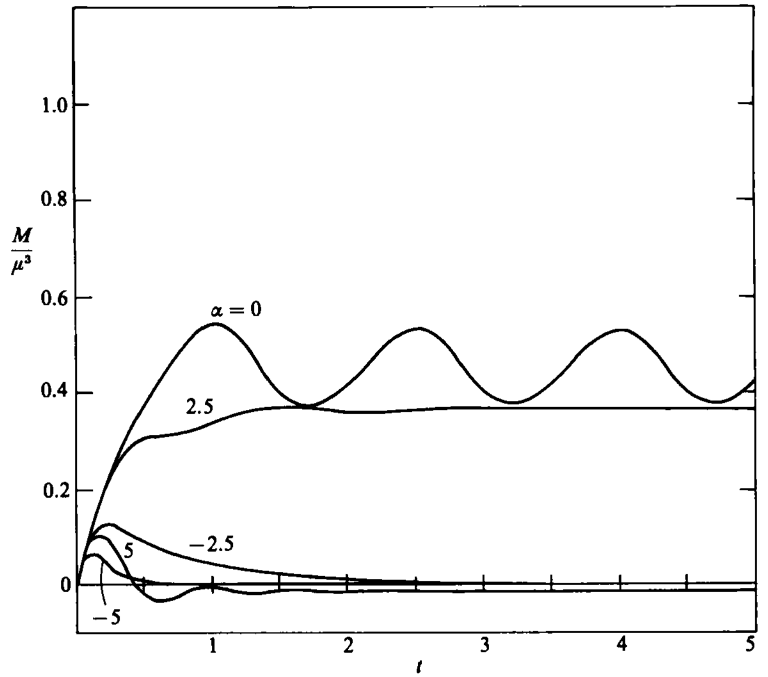

FIGURE 7. Evolution of trimming moment on a slender ship, $\beta = 5$.

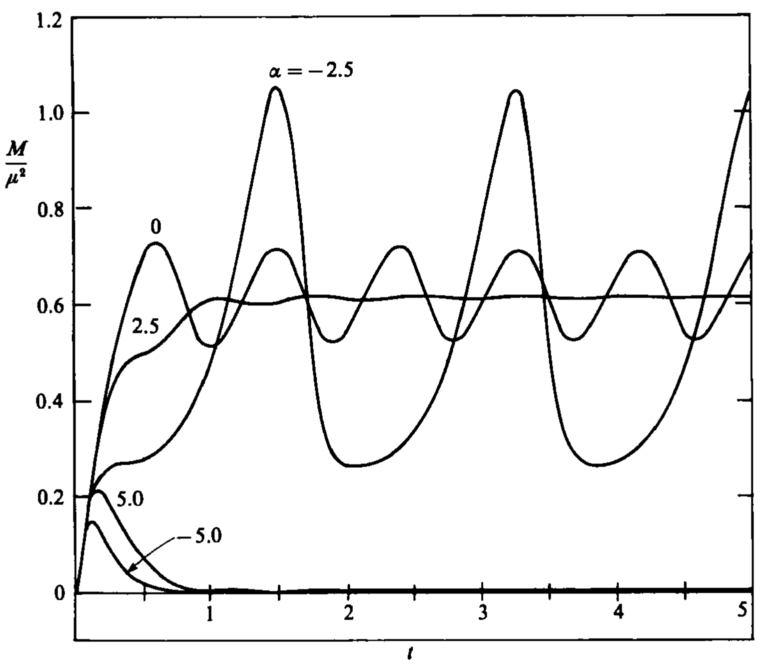

FIGURE 8. Evolution of trimming moment on a slender ship, $\beta = 10$.

waves in the wake, for a low subcritical speed $\alpha = 5$. For a higher subcritical speed $\alpha = 2.5$, the resistance rises higher and approaches a near-steady state, due obviously to the dominance of the downstream waves. At the critical speed, the resistance oscillates around a still higher mean, due to the increasingly large solitons and the downstream waves. The oscillation is in phase with the emission of a new soliton from the ship. At the supercritical speed $\alpha = -2.5$, there is no upstream soliton for the slenderer ship for which the cutoff occurs at about $\alpha = -2.0$, but there are higher solitons for the less slender ship. Therefore, the resistance curve attenuates in time for the former but oscillates even more strongly for the latter, for which the cutoff occurs at about $\alpha = -3.0$. At the high supercritical speed $\alpha = -5.0$, the wave resistance attenuates to zero for both ships.

The vertical lift is shown in figures 5 and 6 for the two ships $\beta = 5$ and 10. It is generally positive (downward) for subcritical speed and negative (upward) for supercritical speed. At and near the critical speed the lift oscillates in time, and is out of phase with respect to the wave resistance. Sufficiently far away from $\alpha = 0$, the lift approaches a steady state corresponding to the disappearance or absence of solitons.

The trimming moment is shown in figures 7 and 8 for $\beta = 5$ and 10, respectively. The features are similar to the wave resistance and are mostly positive, which means bow-up. Near the critical speed the oscillations in time are roughly in phase with the wave resistance.

9. Comparison with experiments

Thus far only Ertekin (1984) has reported ship resistance as a function of time. However, he did not measure the sinkage or trim. We choose instead to compare with the earlier experiments of Graff *et al.* (1964) who performed tests for a destroyer model of length $2L = 3$ m in a tank of width $2W = 10.1$ m. Other gross features of the ship are:

$$2L = 3 \text{ m},\ S_0^* = 0.0215 \text{ m}^2,\ S_B = 0.00142/\mu,$$

$$C_p = 0.64,\ C_v = V/L^3 = 0.0017,\ C_w = 0.78,$$

$$l/2L = 0.00406 \text{ aft} \quad \text{and} \quad r_g/2L = 0.1224.$$

The ship has a blunt (transom) stern; its cross-sections are known at 14 stations and are used in our calculations except at the stern, where we take the cross-sectional area to be zero. Graff *et al.* studied a wide range of depth-to-length ratios ($\mu = 0.10, 0.25, 0.334$ and greater). Since they regarded the results for the $\mu = 0.10$ as unreliable, only the second and third cases are considered here.

We have compared our computed transient wave resistance with the measured but time-averaged residual resistance of Graff *et al.* The latter is the difference between the total resistance and the frictional resistance estimated semi-theoretically. They all showed peaks at about the same ship speed. Because it is well known that accurate separation of frictional and form resistance from the wave resistance is notoriously difficult, comparison with our theory is only of qualitative value and is not presented here.

In figures 9 and 10 we compare the normalized sinkage s and trim θ_T. Note that our theory is for a ship fixed on a carriage and (7.10) and (7.11). While, in the experiments the ship model is only constrained not to move laterally and longitudinally; it is otherwise free to heave and pitch. Since it is a complicated task to construct a complete theory accounting for the ship motion, we follow Tuck (1966)

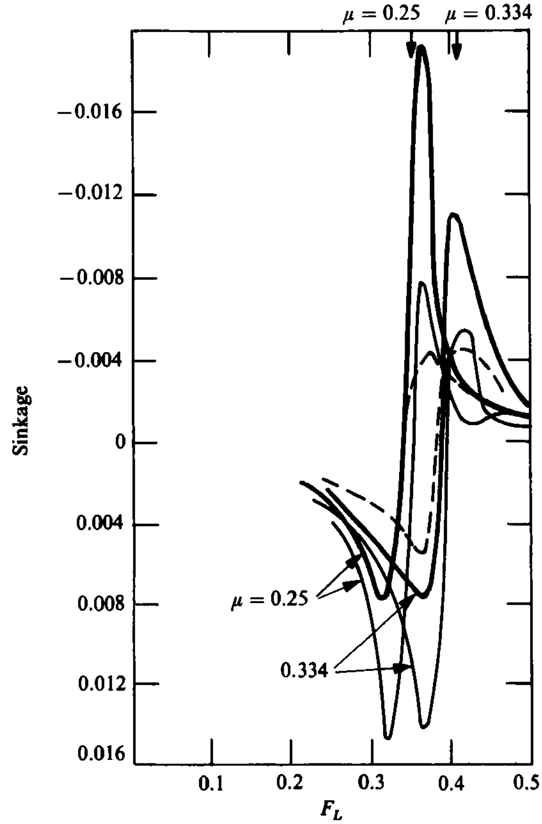

FIGURE 9. Dimensionless sinkage for a destroyer (positive downward). ——, experiment; ——, theoretical first maximum; ----, subsequent minimum; critical speeds are marked by ↓.

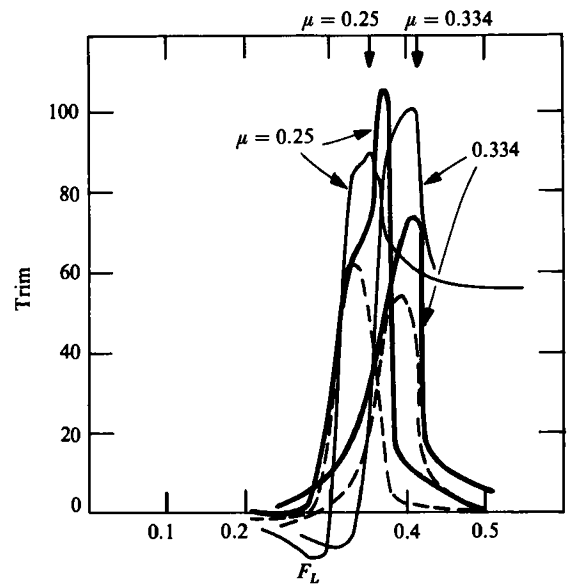

FIGURE 10. Trim angle in minutes for a destroyer (positive if bow up). ——, experiment; ——, theoretical first maximum; ----, subsequent minimum; critical speeds are marked by ↓.

and compare the fixed ship theory with the free ship measurements. Note that the calculated fast maximum and minimum and the measured and time-averaged sinkage and trim depend on the Froude number in very similar ways. They both reach a maximum at a slightly supercritical speed. It seems highly likely that when the dynamic response of a free ship is accounted for, calculations based on this relatively simple nonlinear analysis can yield reliable results for forces on a ship in a wide channel.

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REFERENCES

AKYLAS, T. B. 1984 On the excitation of long nonlinear water waves by a moving pressure distribution. *J. Fluid Mech.* **141**, 455–466.

ERTEKIN, R. C. 1984 Soliton generation by moving disturbances in shallow water. PhD thesis, University of California, Berkeley, 352 pp.

ERTEKIN, R. C., WEBSTER, W. C. & WEHAUSEN, J. V. 1984 Ship-generated solitons. *Proc. 15th Symp. Naval Hydrodyn. Hamburg*, pp. 347–364.

ERTEKIN, R. C., WEBSTER, W. C. & WEHAUSEN, J. V. 1986 Waves caused by a moving disturbance in a shallow channel of finite width. *J. Fluid Mech.* **169**, 275–292.

GRAFF, W., KRACHT, A. & WEINBLUM, G. 1964 Some extension of D. W. Taylor's standard series. *Trans. Soc. Nav. Archit. Mar. Engnrs* **72**, 374–401.

HUANG, D. B., SIBUL, O. J. & WEHAUSEN, J. V. 1983 Ships in very shallow water. *Festkolloquium zur Emeritierung von Karl Wieghardt, Institut fur Schiffbau der Universitat Hamburg, Bericht Nr.* 427 (April 1983), pp. 29–49.

IZUBUCHI, T. & NAGASAWA, S. 1937 Experimental investigation on the influence of water depth upon the resistance of ships. (In Japanese.) *Jap. Soc. Nav. Archit.* **61**, 165–206.

JOHNSON, R. S. 1972 Some numerical solutions of a variable coefficient Korteweg–de Vries equation. *J. Fluid Mech.* **54**, 81–91.

JOUKOWSKI, N. E. 1903 On the wave of translation. *Complete Works 4 ONTI* (1937).

KINOSHITA, M. 1946 On the restricted-water effect on ship resistance. *Jap. Soc. Nav. Archit.* **76** (1954), 173–213.

LEA, G. K. & FELDMAN, J. P. 1972 Transcritical flow past slender ships. *Proc. 8th Symp. Naval Hydrodyn. Paris*, pp. 1527–1651.

MARUO, H. 1948 On the shallow water effect. (In Japanese.) *Jap. Soc. Nav. Archit.* **84** (1952), 1–15.

MARUO, H. & TACHIBANA, T. 1981 An investigation into the sinkage of a ship at the transcritical speed in shallow water. *Jap. Soc. Nav. Archit.* **150**, 181–187.

MEI, C. C. 1986 Radiation of solitons by slender bodies advancing in a shallow channel. *J. Fluid Mech.* **162**, 53–67.

MICHELL, J. H. 1898 The wave resistance of a ship. *Phil. Mag.* **45**, 106–123.

THEWS, J. G. & LANDWEBER, L. 1935 The influence of shallow water on the resistance of a cruiser model. *US Exp. Model Basin, Navy Yard, Washington, DC, Rep.* no. 408.

THEWS, J. G. & LANDWEBER, L. 1936 A thirty-inch model of the SS Clairton in shallow water. *US Exp. Model Basin, Navy Yard, Washington, DC, Rep.* no. 414.

TUCK, E. O. 1966 Shallow-water flows past slender bodies. *J. Fluid Mech.* **26**, 81–95.

WU, D. M. & WU, T. Y. 1982 Three-dimensional nonlinear long waves due to moving surface pressure. *Proc. 14th Symp. Naval Hydrodyn. Ann Arbor*, pp. 103–129.